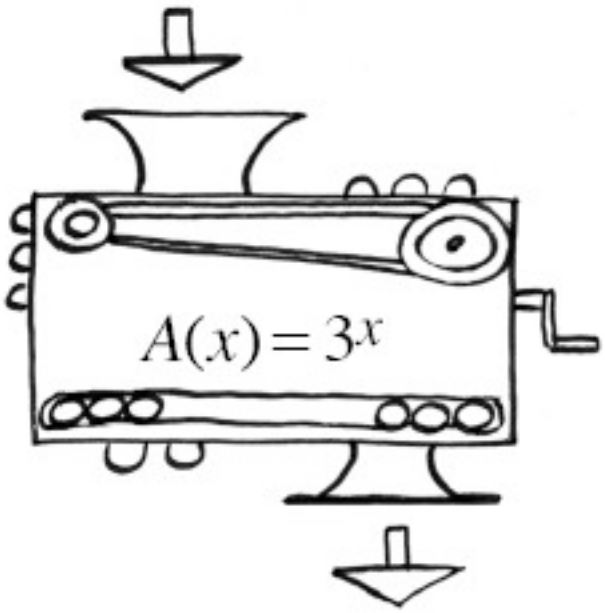
6.1.1 Homework

- **6-7.** Graph $y = \frac{1}{2}x - 3$ and its undoing function on the same set of axes. Hints Help
    1. What is the equation of the undoing function?
    2. Does this graph, including both lines, have a line of symmetry? If so, what is the equation of the line of symmetry?
- **6-8.** Antonio's function machine is shown at right. Hints Help
    1. What is $A(2)$?
    2. If 81 came out, what was dropped in?
    3. If 8 came out, what was dropped in? Be accurate to two decimal places.



- **6-9.** Nossis has been working on his geometry homework and he is almost finished. His last task is to find a solution of $\sin(x) = 0.75$. Nossis cannot figure out what x could be! Explain how he can find a value for x and show that it works. Hints Help
- **6-10.** If $10^x = 10^y$, what is true about x and y? **Justify** your answer. Hints Help
- **6-11.** Solve each of the following equations for x. Hints Help
    1. $\frac{x}{3} = \frac{4}{5}$
    2. $\frac{x}{x+1} = \frac{5}{7}$
    3. $\frac{6}{15} = 2 - \frac{x}{5}$
    4. $\frac{2}{3} + \frac{x}{5} = 6$
- **6-12.** Sketch the solution of this system of inequalities. Hints Help

$$y \geq x^2 - 5$$

$$y \leq -(x-1)^2 + 7$$

- **6-13.** Gary has his function $g(x) = 10^x$ and Amy has her function $a(m) = 10^m$. Hints Help
    1. Each person is going to choose a whole number at random from the numbers 1, 2, 3…10, and substitute it into his or her respective function. After they do this, what is the probability that $g(x) = a(m)$?
    2. Find and simplify an expression for $g(x).a(m)$.
- **6-14.** Jamilla collected data comparing the weight and cost of pieces of sterling silver jewelry. Her data is listed as (weight in ounces, cost in dollars): (5, 44.00), (8.5, 78.50), (12, 112.00), (10, 93.00), (7, 63.50), (9, 83.20). Hints Help
    1. Plot the data on a set of axes.
    2. Use a ruler to draw a line that best approximates the data.
    3. Determine the equation of the line of best fit drawn in (b).
    4. Use your equation to predict the cost of a 50-ounce silver bracelet.
- **6-15.** The angle of elevation of the sun (the angle the rays of sunlight make with the flat ground) at 10:00 a.m. is 29°. At that point, a tree's shadow is 32 feet long. How tall is the tree? Hints Help

6.1.2 Homework

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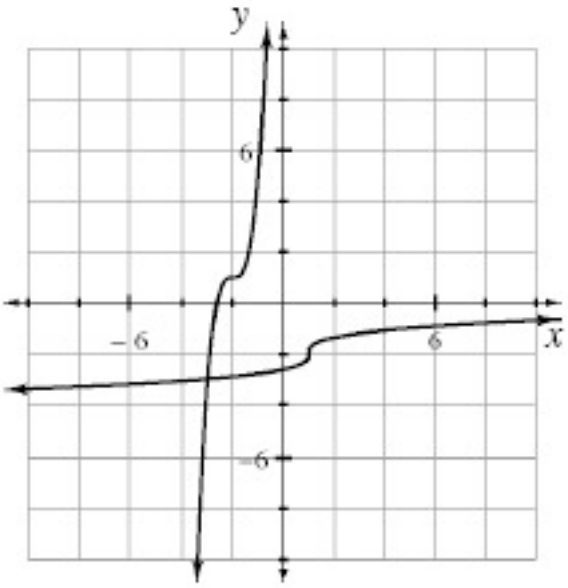
METHODS AND MEANINGS

MATH NOTES



Notation for Inverses

When given a function *f(x)*, the notation for the inverse of the function is $f^{-1}(x)$. For example, if $f(x) = x^3 - 1$ then

$$f^{-1}(x) = \sqrt[3]{x+1}$$

.

Many calculators use this notation to identify the inverse of trigonometric functions. For example the inverse of sin(*x*) is written $\sin^{-1}(x)$.

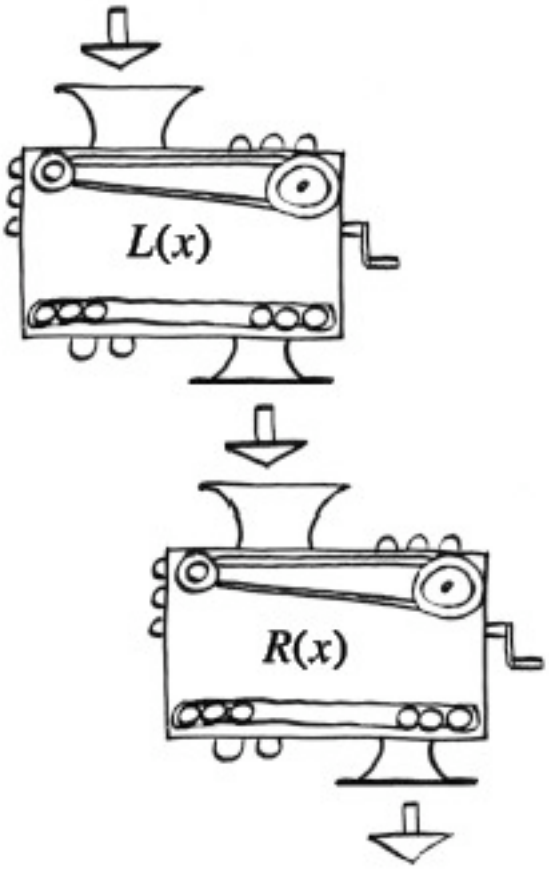
- **6-26.** Make a graph of $f(x) = \frac{1}{2}(x-1)^3$ and then graph its inverse on the same set of axes. Hints Help
- **6-27.** Solve the system of equations below. Hints Help

$$x + y = -3$$
$$2x - y = -6$$
$$3x - 2y + 5z = 16$$

- **6-28.** Solve the equation $3 = 8^x$ for *x*, accurate to the nearest hundredth (two decimal places). Hints Help
- **6-29.** Write the equation of a circle with a center at (−3, 5) that is tangent to the *y*-axis (in other words, it touches the *y*-axis at only one point). Sketching a picture will help. Hints Help
- **6-30.** Perform the indicated operation to simplify each of the following expressions. In some cases, factoring may help you simplify. Hints Help

1. $\frac{(x+2)(x-3)}{(x+1)(x-4)} \cdot \frac{(x+1)}{x(x+2)}$
2. $\frac{x^2+5x+6}{x^2-4} \cdot \frac{4}{x+3}$
3. $\frac{2x}{x+4} + \frac{8}{x+4}$
4. $\frac{x}{x+1} - \frac{1}{x+1}$

- **6-31.** Barnaby's grandfather is always complaining that back when he was a teenager, he used to be able to buy his girlfriend dinner for only $1.50. Hints Help
    1. If that same dinner that Barnaby's grandfather purchased for $1.50 sixty years ago now costs $25.25, and the price has increased exponentially, write an equation that will give you the costs at different times.
    2. How much would you expect the same dinner to cost in 60 years?
- **6-32.** Ever eat a maggot? Guess again! The FDA publishes a list, the Food Defect Action Levels list, which indicates limits for "natural or unavoidable" substances in processed food (*Time*, October 1990). So in 100 g of mushrooms, for instance, the government allows 20 maggots! The average batch of rich and chunky spaghetti sauce has 350 grams of mushrooms. How many maggots does the government allow in a batch? Hints Help
- **6-33.** Lacey and Richens each have their own personal function machines. Lacey's, $L(x)$, squares the input and then subtracts one. Richen's function, $R(x)$, adds 2 to the input and then multiplies the result by three. Hints Help
    1. Write the equations that represent $L(x)$ and $R(x)$.
    2. Lacey and Richens decide to connect their two machines, so that Lacey's output becomes Richens' input. Eventually, what is the output if 3 is the initial input?
    3. What if the order of the machines was changed? Would it change the output? **Justify** your answer.



- **6-34.** Solve the system of equations below. Hints Help
    - $x - 2y = 7$
    - $6y - 3x = 33$
    0. What happened? What does this mean?
    1. What does the solution tell you about the graphs?
- **6-35.** Dana's mother gave her $175 on her sixteenth birthday. "*But you must put it in the bank and leave it there until your eighteenth birthday,*" she told Dana. Dana already had $237.54 in her account, which pays 3.25% annual interest, compounded quarterly. What is the minimum amount of money she will have on her eighteenth birthday if she makes *no* withdrawals before then? **Justify** your answer. Hints Help
- **6-36.** Consider the function $f(x) = \frac{2}{7-x}$. Hints Help
    0. What is $f(7)$?
    1. What is the domain of $f(x)$?
    2. If $g(x) = 2x + 5$, what is $g(3)$?
    3. Now use the output of $g(3)$ as the input for f to calculate $f(g(3))$.
- **6-37.** If $2^{x+4} = 2^{3x-1}$, what is the value of x? Hints Help

6.1.3 Homework

METHODS AND MEANINGS

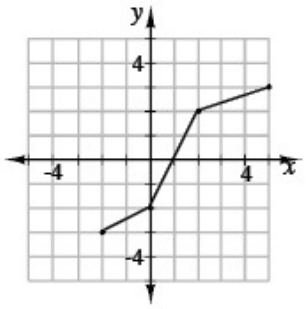
MATH NOTES

Composition of Functions

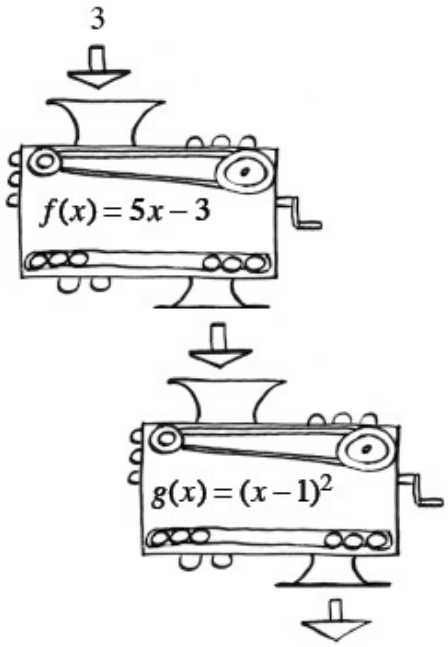
When we stack one function machine on top of another so that the output of the first machine becomes the input of the second, we create a new function, which is a **composition** of the two functions. If the first function is $g(x)$ and the second is $f(x)$, the composition of f and g can be written $f(g(x))$. (Note that the notations $f \circ g$ or $f \circ g(x)$ are used in some texts to denote the same composition.)

Note that the order of the composition matters. In general, the compositions $g(f(x))$ and $f(g(x))$ will be different functions.

- **6-44.** Trejo says that if you know the x-intercepts, y-intercepts, domain, and range of an equation then you automatically know the x-intercepts, y-intercepts, domain, and range for the inverse. Hilary disagrees. She says you know the intercepts but that is all you know for sure. Who is correct? **Justify** your answer. Hints Help
- **6-45.** The function $f(x)$ is represented in the graph below. Draw a graph of its inverse function. Be sure to state the domain and range for both $f(x)$ and $f^{-1}(x)$. Hints Help



- **6-46.** Two function machines, $f(x) = 5x - 3$ and $g(x) = (x - 1)^2$, are shown below. Suppose $f(3)$, (*not* $x = 3$), is dropped into the $g(x)$ machine. This is written as $g(f(3))$. What is this output? Hints Help



- **6-47.** Using the same function machines as in the previous problem, what is $f(g(3))$? Be careful! The result is different from the last one because the *order* in which you use the machines has been switched! With $f(g(3))$, first you find $g(3)$, then you substitute that answer into the machine named f. Hints Help
- **6-48.** This is a Checkpoint for working with integral and rational exponents. Hints Help

Use integer or rational exponents to write each of the following expressions as an exponential expression with a base of x.

1. $\sqrt[5]{x}$
2. $\frac{1}{x^3}$
3. $\sqrt[3]{x^2}$
4. $\frac{1}{\sqrt{x}}$
5. Check your answers by referring to the Checkpoint 11 materials located at the back of your book.

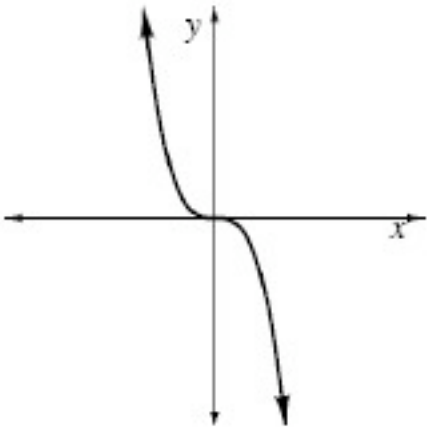
If you needed help to rewrite these expressions correctly, then you need more practice in simplifying expressions with integral or rational exponents. Review Checkpoint 11 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to simplify expressions such as these easily and accurately.

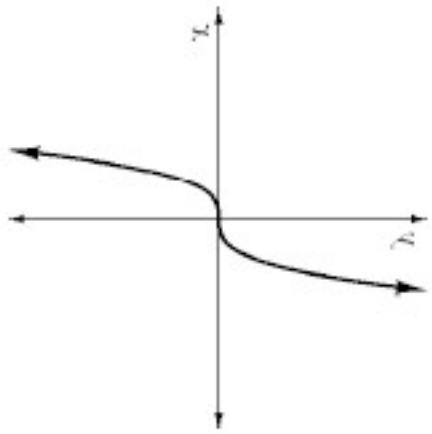
- **6-49.** Solve each of the following equations. Hints Help

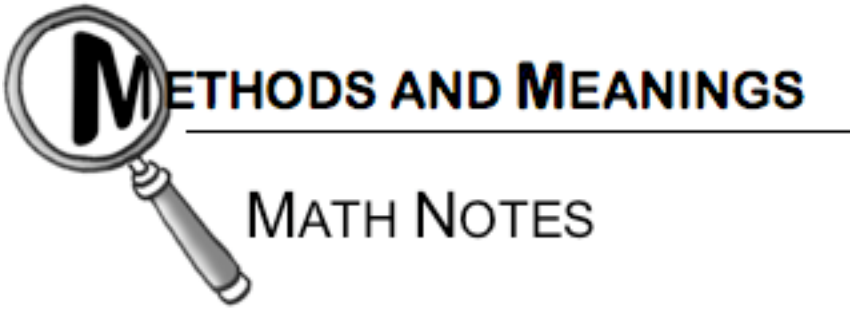
1. $\frac{3x}{5} = \frac{x-2}{4}$
2. $\frac{4x-1}{x} = 3x$
3. $\frac{2x}{5} - \frac{1}{3} = \frac{137}{3}$

4. $\frac{4x-1}{x+1} = x - 1$

- **6-50.** Rebecca thinks that she has found a quick way to graph an inverse of a function. She figures that if you can interchange *x* and *y* to find the inverse, she will interchange the *x*- and *y*-axes by flipping the paper over so that when she looks through the back the *x*-axis is vertical and the *y*-axis is horizontal as shown in the pair of graphs below left. Copy the graph below onto your paper and try her technique. Does it work? If so, do you like this method? Why or why not? Hints Help

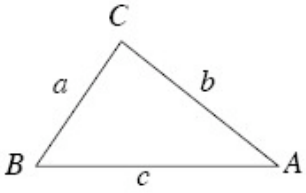






Laws of Sines and Cosines

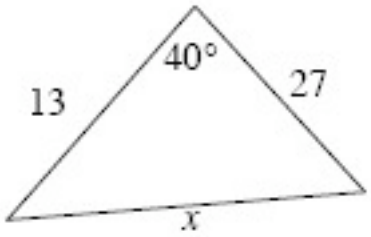
For any uniquely determined triangle, missing sides and angles can be determined by using the **Law of Sines** or the **Law of Cosines**.



Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$

- **6-51.** Find the value of *x*. Refer to the Math Notes box above, for a reminder of the Laws of Sines and Cosines. **Justify** how you know your answer is reasonable. Hints Help

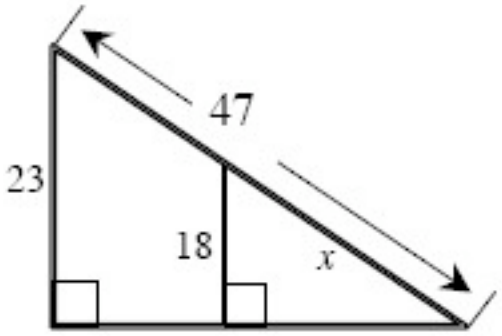


- **6-52.** Complete the square to write $x^2 + y^2 - 4x - 16 = 0$ in graphing form and sketch the graph. Hints Help
- **6-53.** Perform each operation below and simplify your results. Hints Help

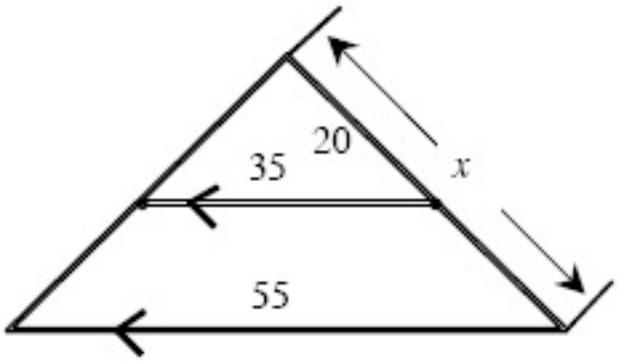
1. $\dfrac{x^2+4x+3}{x^2+3x} \cdot \dfrac{3x}{x+1}$
2. $\dfrac{y^2}{y+4} - \dfrac{16}{y+4}$
3. $\dfrac{x^2+x}{x^2-4x-5} \div \dfrac{3x^2}{x-5}$
4. $\dfrac{x^2-6x}{x^2-4x+4} + \dfrac{4x}{x^2-4x+4}$

6.2.1 Homework

- **6-59.** In problem 6-55, you looked at the inverse of $y = 3^x$. Finish **investigating** this function. Hints Help
- **6-60.** Amanda wants to showcase her favorite function: $f(x) = 1 + \sqrt{x+5}$. She has built a function machine that performs these operations on the input values. Her brother Eric is always trying to mess up Amanda's stuff, so he created the inverse of *f(x)*, called it *e(x)*, and programmed it into a machine. Hints Help
    1. What is Eric's equation for his function $e(x)$?
    2. What happens if the two machines are pushed together? What is $e(f(-4))$? Explain why this happens.
    3. If $f(x)$ and $e(x)$ are graphed on the same set of axes, what would be true about the two graphs?
    4. Draw the two graphs on the same set of axes. Be sure to show clearly the restricted domain and range of Amanda's function.
- **6-61.** Sketch the graph of $y + 3 = 2^x$. Hints Help
    1. What are the domain and range of this function?
    2. Does this function have a line of symmetry? If so, what is it?
    3. What are the *x*- and *y*-intercepts?
    4. Change the equation so that the graph of the new equation has no *x*-intercepts.
- **6-62.** Solve for *x* in the following diagrams. Hints Help



1.



2.

- **6-63.** Sketch square *ABCD* on your paper, then randomly choose a point on $\overline{AB}$ and label it *X*. Draw $\overline{XC}$ and $\overline{XD}$ to form *ΔXCD*. If a dart is thrown and lands inside the square, what is the probability that it landed inside *ΔXCD*? Does it matter where you place *X* on $\overline{AB}$? Hints Help
- **6-64.** A woman plans to invest *x* dollars. Her investment counselor advises her that a safe plan is to invest 30% of that money in bonds and 70% in low risk stocks. The bonds currently have a simple interest rate of 7% and the stock has a dividend rate (like simple interest) of 9%. Hints Help
    1. Write an expression for the annual income that will come from the bond investment.
    2. Write an expression for the annual income that will come from the stock investment.
    3. Write an equation and solve it to find out how much the client needs to invest to have an annual income of $5,000.

- **6-65.** Some of the following algebraic fractions have common denominators and some do not. Add or subtract the expressions and simplify, if possible. Hints Help

1. $\frac{3}{(x-4)(x+1)} + \frac{6}{x+1}$

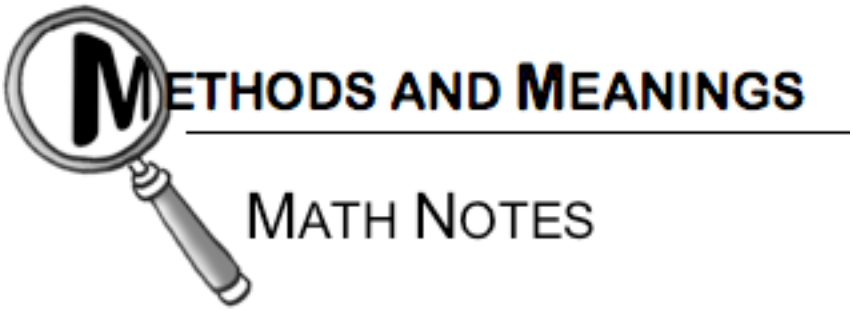
2. $\frac{5}{2(x-5)} + \frac{3x}{x-5}$

3. $\frac{x}{x^2-x-2} - \frac{2}{x^2-x-2}$

4. $\frac{x+2}{x^2-9} - \frac{1}{x+3}$

- **6-66.** Sketch the solution to this system of inequalities. Hints Help

6.2.2 Homework



Logarithms and Their Notation

A **logarithm** (called a "Log" for short) is an exponent. An expression in logarithmic form, such as $\log_2(32)$, is read, "*the log, base 2, of 32*." To evaluate log expressions, think of the exponent: $\log_2(32) = 5$, because the exponent needed for base 2 to become 32 is 5.

An equation in logarithmic form is equivalent to another equation in exponential form, as shown below. This conversion helps show why (based on an $x \rightarrow y$ interchange) $y = \log_b(x)$ and $y = b^x$ are inverse functions.

$$y = \log_b(x) \qquad b^y = x$$

- **6-72.** Let $y = \log_2(x)$. Rewrite the equation so that it begins with $x =$. Think about how you defined $y = \log_2(x)$ if you get stuck. Put a large box around both equations. Do the two equations look the same? Do the two equations mean the same thing? Are they equivalent? How do you know? This is very important. Think about it, and write a clear explanation. Hints Help
- **6-73.** Every exponential equation has an equivalent logarithmic form and every logarithmic equation has an equivalent exponential form. For example,

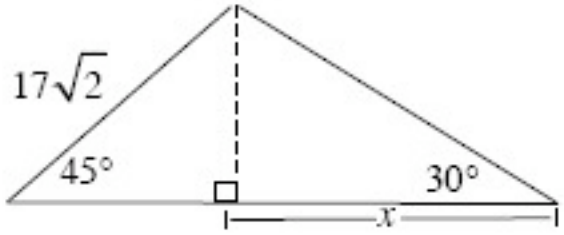
$$\underset{\text{base}}{4}\overset{\text{exponent}}{^{3}} = 64 \quad \text{is equivalent to} \quad 3 = \underset{\text{base}}{\log_4} 64 \quad (\text{exponent})$$

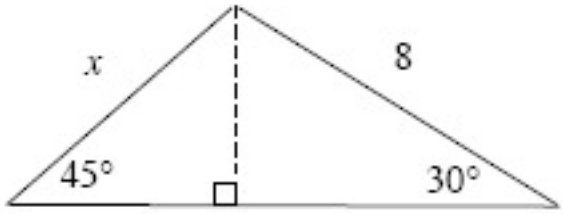
Copy the table shown below and fill in the missing form in each row. Hints Help

	Exponential Form	Logarithmic Form

a.	$y = 5^x$	
b.		$y = \log_7(x)$
c.	$8^x = y$	
d.	$A^K = C$	
e.		$K = \log_A(C)$
f.		$\log_{1/2}(K) = N$

- **6-74.** If $x = 7^y$, how would you write this equation in y-form? Explain. Hints Help
- **6-75.** Find the value of x in the equation $2^x = 3$. Be accurate to three decimal places. Hints Help
- **6-76.** Although the quadratic formula always works as a **strategy** to solve quadratic equations, for many problems it is not the most efficient method. Sometimes it is faster to factor or complete the square or even just "out-think" the problem. For each equation below, choose the method you think is most efficient to solve the equation and explain your reason. **Note that you do *not* actually need to solve the equation.** Hints Help
    1. $x^2 + 7x - 8 = 0$
    2. $(x + 2)^2 = 49$
    3. $5x^2 - x - 7 = 0$
    4. $x^2 + 4x = -1$
- **6-77.** If $10^{3x} = 10^{(x-8)}$, solve for x. Show that your solution works by checking your answer. Hints Help
- **6-78.** Find the value of x in each diagram below. Hints Help

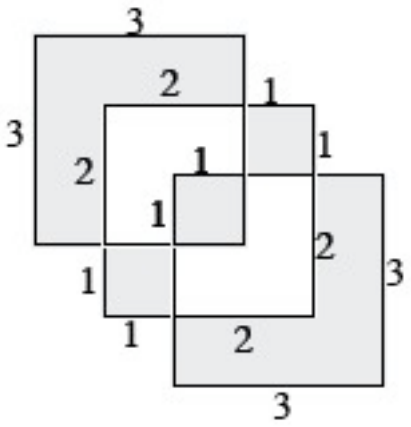
1. 

2. 

- **6-79.** Consider the function defined by inputs that are the length of the radii of a circle, and the outputs are the areas of those circles. Write the rule for this function and **investigate** it completely. Hints Help
- **6-80.** Consider the equation $y = (x + 6)^2 - 7$. Hints Help
    1. Explain completely how to get a good sketch of the graph of $y = (x + 6)^2 - 7$.
    2. Explain how to change the original graph to represent the graph of $y = (x + 6)^2 + 2$.
    3. Given the original graph, how can you get the graph of $y = \left|(x+6)^2 - 7\right|$?
    4. Restrict the domain of the original parabola to $x \geq -6$ and graph its inverse function.
    5. What would be the equation for the inverse function if you restricted the domain to $x \geq -6$?

6.2.3 Homework

- **6-84.** Write the equation of an increasing exponential function that has a horizontal asymptote at $y = 15$. Hints Help
- **6-85.** If a point inside the figure below is chosen at random, what is the probability that it is in the shaded region? Hints Help



- **6-86.** Solve for n: $n^3 = 49$. Hints Help
- **6-87.** A circle has the equation $x^2 + (y + 2)^2 = r^2$. If the circle is shifted 2 units to the left, 5 units up, and the radius is doubled, what will its new equation be? Hints Help
- **6-88.** On Wednesdays at Tara's Taqueria four tacos are the same price as three burritos. Last Wednesday the Lunch Bunch ordered five tacos and six burritos, and their total bill was $8.58 (with no tax or drinks included). Nobody in the Lunch Bunch can remember the cost of one of Tara's tacos. Help them figure it out. Hints Help
- **6-89.** Graph the two functions below on the same set of axes. Hints Help

$$y = 3(2^x)$$
$$y = 3(2^x) + 10$$

1. How do the two graphs compare?
2. Suppose the first equation is $y = km^x$ and the graph is shifted up b units. What is the new equation?

- **6-90.** Solve each equation or inequality. Hints Help

1. $|x - 1| = 9$
2. $2|x + 1| + 3 = 9$
3. $|x - 1| < 3$
4. $|x + 5| \geq 8$

- **6-91.** For each of the following rational expressions, add or subtract, then simplify. Hints Help

1. $\frac{2-x}{x+4} + \frac{3x+6}{x+4}$
2. $\frac{3}{(x+2)(x+3)} + \frac{x}{(x+2)(x+3)}$

3. $\frac{3}{x-1} - \frac{2}{x-2}$

4. $\frac{8}{x} - \frac{4}{x+2}$

- **6-92.** Each step of a simplification process must be **justifiable** using the properties of algebra. Hints Help
  1. Examine the **justification** for each step in the simplification below.

     Given expression: $2(x+\frac{3}{x})-\frac{4}{x}$

     Step 1: $2x+\frac{6}{x}-\frac{4}{x}$ Distributive Property

     Step 2: $\frac{2x^2}{x}+\frac{6}{x}-\frac{4}{x}$ Multiplicative Identity ($1 \cdot a = a$)

     Step 3: $\frac{1}{x}(2x^2+6-4)$ Distributive Property

     Step 4: $\frac{2x^2+6-4}{x}$ Definition of Division ($a \div b = a(\frac{1}{b})$)

     Step 5: $\frac{2x^2+2}{x}$ Associative Property of Addition
  2. Use the properties of algebra to **justify** each step in simplifying the expression in part (d) of problem 6-91.

6.2.4 Homework

- **6-96.** Last night, while on patrol, Agent 008 came upon a spaceship! He hid behind a tree and watched a group of little space creatures carry all sorts of equipment out of the ship. But suddenly, he sneezed. The creatures jumped back into their ship and sped off into the night. 008 noticed that they had dropped something so he went to pick it up. It was a calculator! What a great find. He noticed that it had a LOG button, but he noticed something interesting: log 10 did not equal 1! With this calculator, log10 ≈ 0.926628408. He tried some more: log 100 ≈ 1.853256816 and log 1000 ≈ 2.779885224. Hints Help
    1. What base do the space creatures work in? Explain how you can tell.
    2. How many fingers do you think the space creatures have?
- **6-97.** Copy these equations and solve for x. You should be able to do all these problems without a calculator. Hints Help

    1. $\log_x(25) = 1$
    2. $x = \log_3(9)$
    3. $3 = \log_7(x)$
    4. $\log_3(x) = \frac{1}{2}$
    5. $3 = \log_x(27)$
    6. $\log_{10}(10000) = x$
- **6-98.** Is log(0.3) greater than or less than one? **Justify** your anwer. Hints Help

- **6-99.** Solve $1.04^x = 2$. Your answer should be accurate to three decimal places. Hints Help
- **6-100.** Perform each operation below and simplify your results. Hints Help

    1. $\frac{x^2+5x+6}{x^2-4x} \cdot \frac{4x}{x+2}$
    2. $\frac{x^2-2x}{x^2-4x+4} + \frac{4x^2}{x-2}$
- **6-101.** Solve the following inequalities. Hints Help
    1. $x^2 - 2x < 3$
    2. $3x - x^2 \le 2$
- **6-102.** Solve for m: $m^5 = 50$. Hints Help
- **6-103.** Is it true that $\log_3(2) = \log_2(3)$? **Justify** your answer. Hints Help

- **6-104.** Consider the general form of an exponential function: $y = ab^x$. Hints Help
    1. Solve for a.
    2. Solve for b.
- **6-105.** Make a sketch of a graph that is a decreasing exponential function with the x-axis as the horizontal asymptote. Then make a similar sketch, but this time with the line $y = 5$ as the horizontal asymptote. Hints Help